

User's Guide of a Sequent Prover (seqprover)

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1 Introduction

This program finds a cut-free proof of a given sequent (however, cut rules are used when axioms are given). That is, it creates a proof from bottom to top by applying applicable rules. The CL-X system used here does not have structural rules explicitly (see Appendix A). Therefore, the resulting proof does not include structural rules.

2 Notations

Syntax follows SICStus Prolog. But some operators are redefined as follows.

```
:- op(1200, xfx, [ -->, <--> ] ).
:- op( 900, xfy, [ -> ] ).
:- op( 850, xfy, [ /\, \/ ] ).
:- op( 800, fy, [ ~ ] ).
:- op( 750, xfy, [ @, # ] ).
```

Smaller number means stronger (narrower) scope. Also, `xfx` means an infix operator, `xfy` means a right associative operator (e.g. $A + B + C = A + (B + C)$), `fy` means a prefix operator.

Please follow the following notation to represent variables, etc.

- Variables: Please use Prolog variables (uppercase letters): `X`, `Y`, `X0`, `...`, etc.
- Constants: Please use Prolog atoms (lowercase letters): `a`, `b`, `1`, `...`, etc.
- Function and Predicate symbols: Please use Prolog functor symbols (lowercase letters): `a`, `b`, `c`, `...`, etc.

Logical connectives are written as follows:

Notation used here	Standard notation
<code>top</code>	\top
<code>bot</code>	\perp
<code>~p</code>	$\neg P$
<code>p/\q</code>	$P \wedge Q$
<code>p\/q</code>	$P \vee Q$
<code>p->q</code>	$P \supset Q$
<code>X@p</code>	$\forall x.P$
<code>X#p</code>	$\exists x.P$

Sequents are written as follows:

Notation used here	Standard notation
$A_1, A_2, \dots, A_m \dashrightarrow B_1, B_2, \dots, B_n$	$A_1, A_2, \dots, A_m \longrightarrow B_1, B_2, \dots, B_n \quad (m, n > 0)$
$A_1, A_2, \dots, A_m \dashrightarrow \square$	$A_1, A_2, \dots, A_m \longrightarrow \quad (m > 0)$
$\square \dashrightarrow B_1, B_2, \dots, B_n$	$\longrightarrow B_1, B_2, \dots, B_n \quad (n > 0)$
$\square \dashrightarrow \square$	\longrightarrow

A CL-X

$$\begin{array}{c}
\overline{\Gamma_1, A, \Gamma_2 \longrightarrow \Delta_1, A, \Delta_2} \text{ (Ax)} \\
\\
\frac{\Gamma_1, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \top, \Gamma_2 \longrightarrow \Delta} \text{ (L}\top\text{)} \qquad \qquad \qquad \overline{\Gamma \longrightarrow \Delta_1, \top, \Delta_2} \text{ (R}\top\text{)} \\
\\
\overline{\Gamma_1, \perp, \Gamma_2 \longrightarrow \Delta} \text{ (L}\perp\text{)} \qquad \qquad \qquad \frac{\Gamma \longrightarrow \Delta_1, \Delta_2}{\Gamma \longrightarrow \Delta_1, \perp, \Delta_2} \text{ (R}\perp\text{)} \\
\\
\frac{\Gamma_1, \Gamma_2 \longrightarrow A, \Delta}{\Gamma_1, \neg A, \Gamma_2 \longrightarrow \Delta} \text{ (L}\neg\text{)} \qquad \qquad \qquad \frac{A, \Gamma \longrightarrow \Delta_1, \Delta_2}{\Gamma \longrightarrow \Delta_1, \neg A, \Delta_2} \text{ (R}\neg\text{)} \\
\\
\frac{\Gamma_1, A, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \wedge B, \Gamma_2 \longrightarrow \Delta} \text{ (L}\wedge\text{)} \qquad \qquad \qquad \frac{\Gamma \longrightarrow \Delta_1, A, \Delta_2 \quad \Gamma \longrightarrow \Delta_1, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \wedge B, \Delta_2} \text{ (R}\wedge\text{)} \\
\\
\frac{\Gamma_1, A, \Gamma_2 \longrightarrow \Delta \quad \Gamma_1, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \vee B, \Gamma_2 \longrightarrow \Delta} \text{ (L}\vee\text{)} \qquad \qquad \qquad \frac{\Gamma \longrightarrow \Delta_1, A, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \vee B, \Delta_2} \text{ (R}\vee\text{)} \\
\\
\frac{\Gamma_1, \Gamma_2 \longrightarrow A, \Delta \quad \Gamma_1, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \supset B, \Gamma_2 \longrightarrow \Delta} \text{ (L}\supset\text{)} \qquad \qquad \qquad \frac{A, \Gamma \longrightarrow \Delta_1, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \supset B, \Delta_2} \text{ (R}\supset\text{)} \\
\\
\frac{\Gamma_1, A[t/x], \forall x.A, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \forall x.A, \Gamma_2 \longrightarrow \Delta} \text{ (L}\forall\text{)} \qquad \qquad \qquad \frac{\Gamma \longrightarrow \Delta_1, A[y/x], \Delta_2}{\Gamma \longrightarrow \Delta_1, \forall x.A, \Delta_2} \text{ (R}\forall\text{)} \\
\\
\frac{\Gamma_1, A[y/x], \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \exists x.A, \Gamma_2 \longrightarrow \Delta} \text{ (L}\exists\text{)} \qquad \qquad \qquad \frac{\Gamma \longrightarrow \Delta_1, A[t/x], \exists x.A, \Delta_2}{\Gamma \longrightarrow \Delta_1, \exists x.A, \Delta_2} \text{ (R}\exists\text{)}
\end{array}$$

Here, y is not free in the lower sequent in (R \forall) and (L \exists).