# User's Guide of a Sequent Prover (seqprover)

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April 22, 2003

## 1 Introduction

This program finds a cut-free proof of a given sequent (however, cut rules are used when axioms are given). That is, it creates a proof from bottom to up by applying applicable rules. The CL-X system used here does not have structural rules explicitly (see Appendix A). Therefore, the resulting proof does not include structurual rules.

## 2 Notations

Syntax follows SICStus Prolog. But some operators are redefined as follows.

```
:- op(1200, xfx, [ -->, <--> ]).
:- op( 900, xfy, [ -> ]).
:- op( 850, xfy, [ /\, \/ ]).
:- op( 800, fy,  [ ~ ]).
:- op( 750, xfy, [ @, # ]).
```

Smaller number means stronger (narrower) scope. Also, `xfx` means an infix operator, `xfy` means a right associative operator (e.g. $A + B + C = A + (B + C)$), `fy` means a prefix operator.

Please follow the following notation to represent variables, etc.

- Variables: Please use Prolog variables (uppercase letters): `X`, `Y`, `X0`, . . . , etc.
- Constants: Please use Prolog atoms (lowercase letters): `a`, `b`, `1`, . . . , etc.
- Function and Predicate symbols: Please use Prolog functor symbols (lowercase letters): `a`, `b`, `c`, . . . , etc.

Logical connectives are written as follows:

| Notation used here | Standard notation |
|---|---|
| `top` | $\top$ |
| `bot` | $\bot$ |
| `~p` | $\neg P$ |
| `p/\q` | $P \wedge Q$ |
| `p\/q` | $P \vee Q$ |
| `p->q` | $P \supset Q$ |
| `X@p` | $\forall x.P$ |
| `X#p` | $\exists x.P$ |

Sequents are written as follows:

| Notation used here | Standard notation | |
|---|---|---|
| `A1,A2,...,Am --> B1,B2,...,Bn` | $A_1, A_2, \ldots, A_m \longrightarrow B_1, B_2, \ldots, B_n$ | $(m, n > 0)$ |
| `A1,A2,...,Am --> []` | $A_1, A_2, \ldots, A_m \longrightarrow$ | $(m > 0)$ |
| `[] --> B1,B2,...,Bn` | $\longrightarrow B_1, B_2, \ldots, B_n$ | $(n > 0)$ |
| `[] --> []` | $\longrightarrow$ | |

# A CL-X

$$\frac{}{\Gamma_1, A, \Gamma_2 \longrightarrow \Delta_1, A, \Delta_2}\ (\mathrm{Ax})$$

$$\frac{\Gamma_1, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \top, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\top) \qquad \frac{}{\Gamma \longrightarrow \Delta_1, \top, \Delta_2}\ (\mathrm{R}\top)$$

$$\frac{}{\Gamma_1, \bot, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\bot) \qquad \frac{\Gamma \longrightarrow \Delta_1, \Delta_2}{\Gamma \longrightarrow \Delta_1, \bot, \Delta_2}\ (\mathrm{R}\bot)$$

$$\frac{\Gamma_1, \Gamma_2 \longrightarrow A, \Delta}{\Gamma_1, \neg A, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\neg) \qquad \frac{A, \Gamma \longrightarrow \Delta_1, \Delta_2}{\Gamma \longrightarrow \Delta_1, \neg A, \Delta_2}\ (\mathrm{R}\neg)$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \wedge B, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\wedge) \qquad \frac{\Gamma \longrightarrow \Delta_1, A, \Delta_2 \quad \Gamma \longrightarrow \Delta_1, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \wedge B, \Delta_2}\ (\mathrm{R}\wedge)$$

$$\frac{\Gamma_1, A, \Gamma_2 \longrightarrow \Delta \quad \Gamma_1, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \vee B, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\vee) \qquad \frac{\Gamma \longrightarrow \Delta_1, A, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \vee B, \Delta_2}\ (\mathrm{R}\vee)$$

$$\frac{\Gamma_1, \Gamma_2 \longrightarrow A, \Delta \quad \Gamma_1, B, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, A \supset B, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\supset) \qquad \frac{A, \Gamma \longrightarrow \Delta_1, B, \Delta_2}{\Gamma \longrightarrow \Delta_1, A \supset B, \Delta_2}\ (\mathrm{R}\supset)$$

$$\frac{\Gamma_1, A[t/x], \forall x.A, \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \forall x.A, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\forall) \qquad \frac{\Gamma \longrightarrow \Delta_1, A[y/x], \Delta_2}{\Gamma \longrightarrow \Delta_1, \forall x.A, \Delta_2}\ (\mathrm{R}\forall)$$

$$\frac{\Gamma_1, A[y/x], \Gamma_2 \longrightarrow \Delta}{\Gamma_1, \exists x.A, \Gamma_2 \longrightarrow \Delta}\ (\mathrm{L}\exists) \qquad \frac{\Gamma \longrightarrow \Delta_1, A[t/x], \exists x.A, \Delta_2}{\Gamma \longrightarrow \Delta_1, \exists x.A, \Delta_2}\ (\mathrm{R}\exists)$$

Here, $y$ is not free in the lower sequent in (R∀) and (L∃).